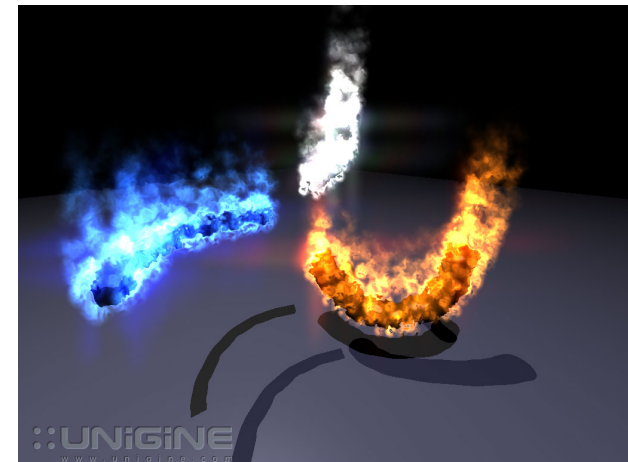




# Virtual Reality & Physically-Based Simulation Particle Systems



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[cgvr.cs.uni-bremen.de](http://cgvr.cs.uni-bremen.de)

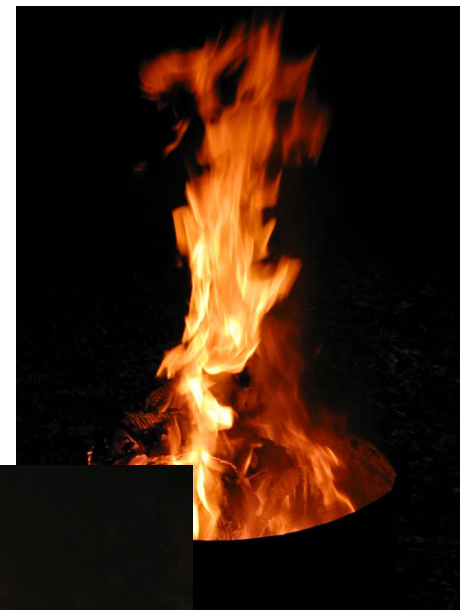




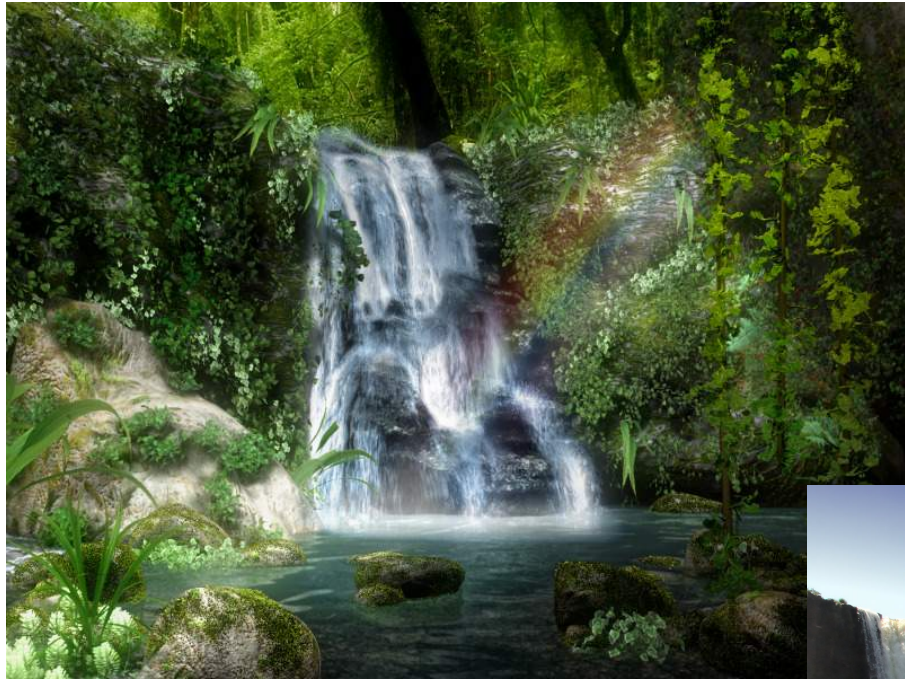
# How to Model/Simulate/Render Natural Phenomena?





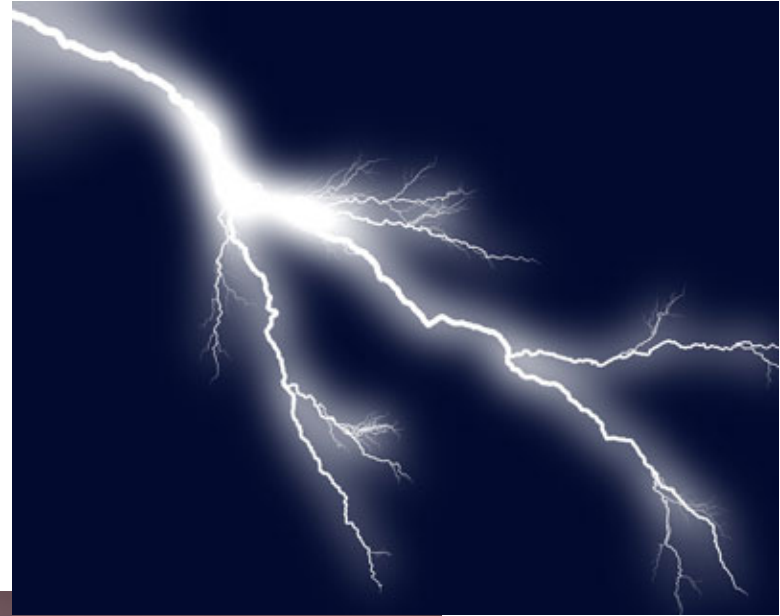














# Dynamics of a Point Mass

- Definition **Particle**:

A particle is ideal point with a mass  $m$  and a velocity  $\mathbf{v}$ .

It does not process an orientation.

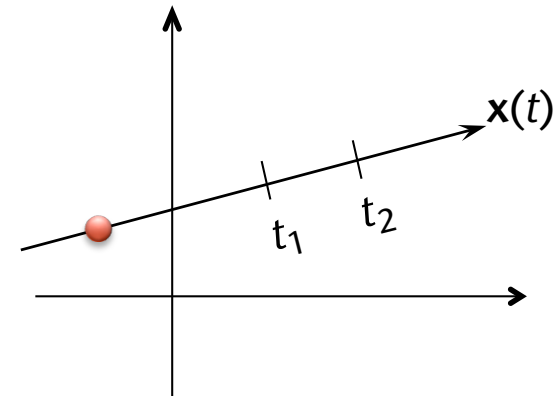
- Path of a particle =  $\mathbf{x}(t)$

- Velocity:

$$\mathbf{v} = \frac{\text{distance}}{\text{time}} = \frac{\mathbf{x}(t_2) - \mathbf{x}(t_1)}{t_2 - t_1}$$

- Unit:  $m/s$

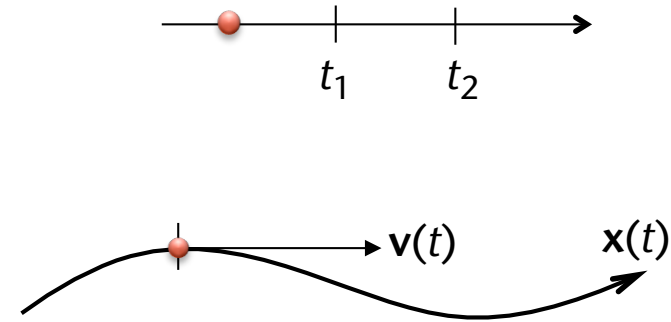
- Note: velocity of particle = **vector**
    - position of particle = **point!**



- The momentary velocity:

$$\mathbf{v}(t_1) = \lim_{t_2 \rightarrow t_1} \frac{\mathbf{x}(t_2) - \mathbf{x}(t_1)}{t_2 - t_1}$$

$$= \frac{d}{dt} \mathbf{x}(t_1) = \dot{\mathbf{x}}(t_1)$$




- Examples:

- Point moves on a circular path  $\rightarrow \|\dot{\mathbf{x}}\|$  is constant
- Point accelerates on a straight line  $\rightarrow \frac{\dot{\mathbf{x}}}{\|\dot{\mathbf{x}}\|}$  is constant

- Acceleration at some point in time :

$$\mathbf{a}(t) = \frac{d}{dt} \mathbf{v}(t) = \dot{\mathbf{v}}(t) = \frac{\mathbf{F}(t)}{m}$$


 Newtons 2. Law



# Euler Integration

- Given: a particle of mass  $m$ ;  
and a force  $\mathbf{F}(t)$  that acts on the particle over time
- Wanted: the path  $\mathbf{x}(t)$  of the particle
- The analytical approach:

$$\mathbf{v}(t) = \mathbf{v}_0 + \int_{t_0}^t \mathbf{a}(t) dt$$

$$\mathbf{x}(t) = \mathbf{x}_0 + \int_{t_0}^t \mathbf{v}(t) dt$$

- Discretization and linearization yields:

$$v^{t+1} = v^t + a^t \cdot \Delta t$$

$$x^{t+1} = x^t + v^t \cdot \Delta t$$

or

$$x^{t+1} = x^t + \frac{v^t + v^{t+1}}{2} \Delta t$$

*(approx. midpoint method)*

- The (physical) momentary state of a particle is described completely by

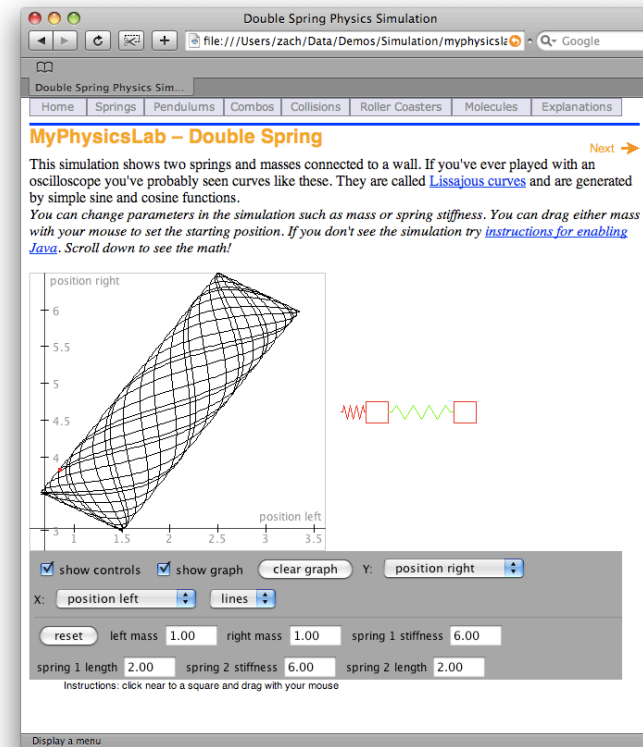
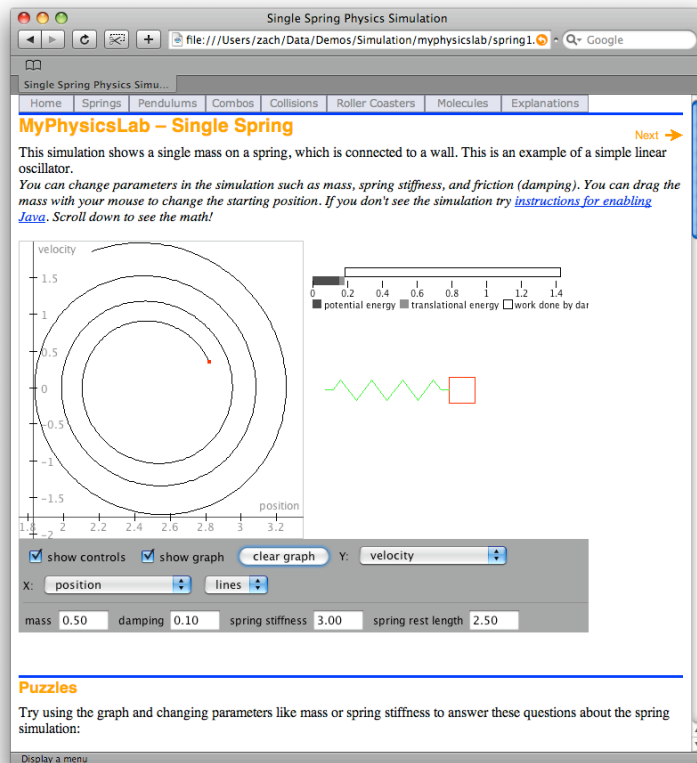
$$\begin{aligned}\mathbf{q} &= (\mathbf{x}, \mathbf{v}) = (x_1, x_2, x_3, v_1, v_2, v_3) \\ &= (x_1, x_2, x_3, \dot{x}_1, \dot{x}_2, \dot{x}_3) \in \mathbb{R}^6\end{aligned}$$

- The space of all possible states is called *phase space*
- The dimension is  $6n$ ,  $n$  = number of particles
- The motion of a particle in phase space:

$$\dot{\mathbf{q}} = (\dot{x}_1, \dot{x}_2, \dot{x}_3, \dot{v}_1, \dot{v}_2, \dot{v}_3) = \left( v_1, v_2, v_3, \frac{\mathbf{f}_1}{m}, \frac{\mathbf{f}_2}{m}, \frac{\mathbf{f}_3}{m} \right)$$



- Example for a particle that can move only along the X axis and that is held in a resting position by a spring:

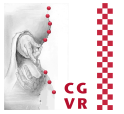


[www.myphysicslab.com](http://www.myphysicslab.com)



# Laplace's Demon

[1814]



- *Causal/scientific determinism* postulates: if someone (the demon) knows the precise location and momentum of every atom in the universe, their future values for any given time can be calculated from the laws of classical mechanics.
- Consequences:
  - Everything would be pre-determined
  - There would be no free will
- Fortunately, such a demon is impossible





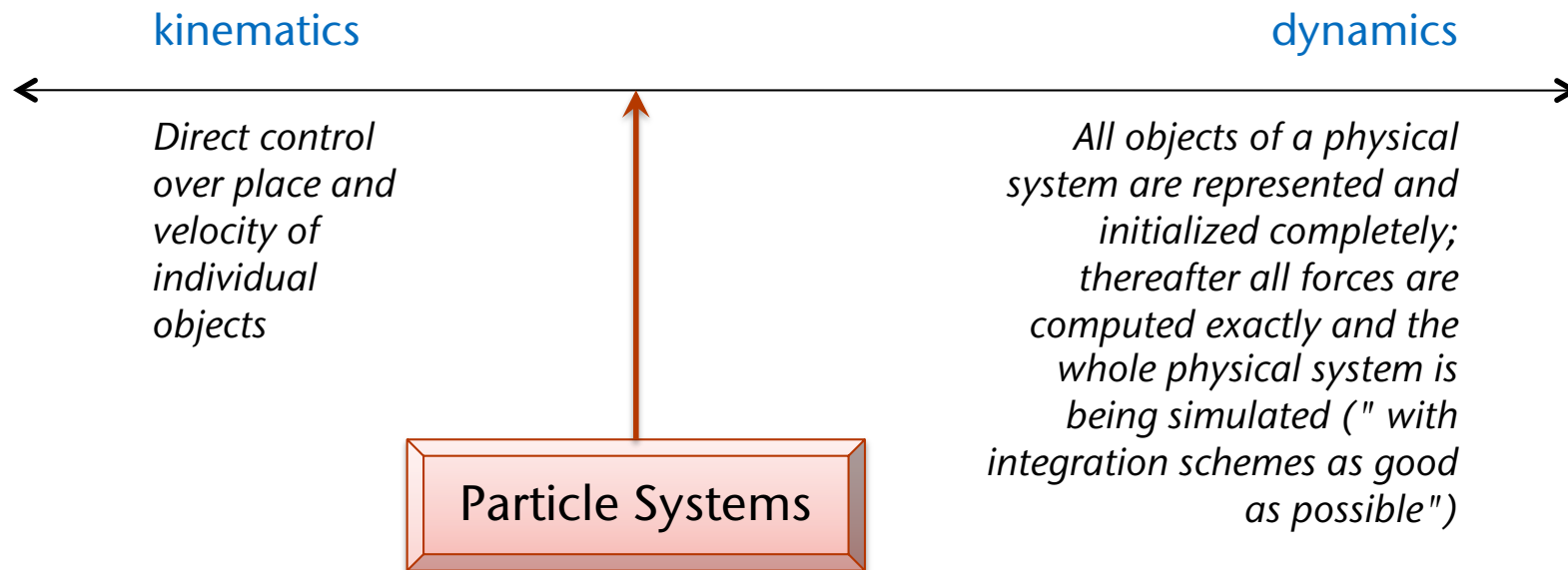
# Kinematics vs Dynamics

- Technical terms:

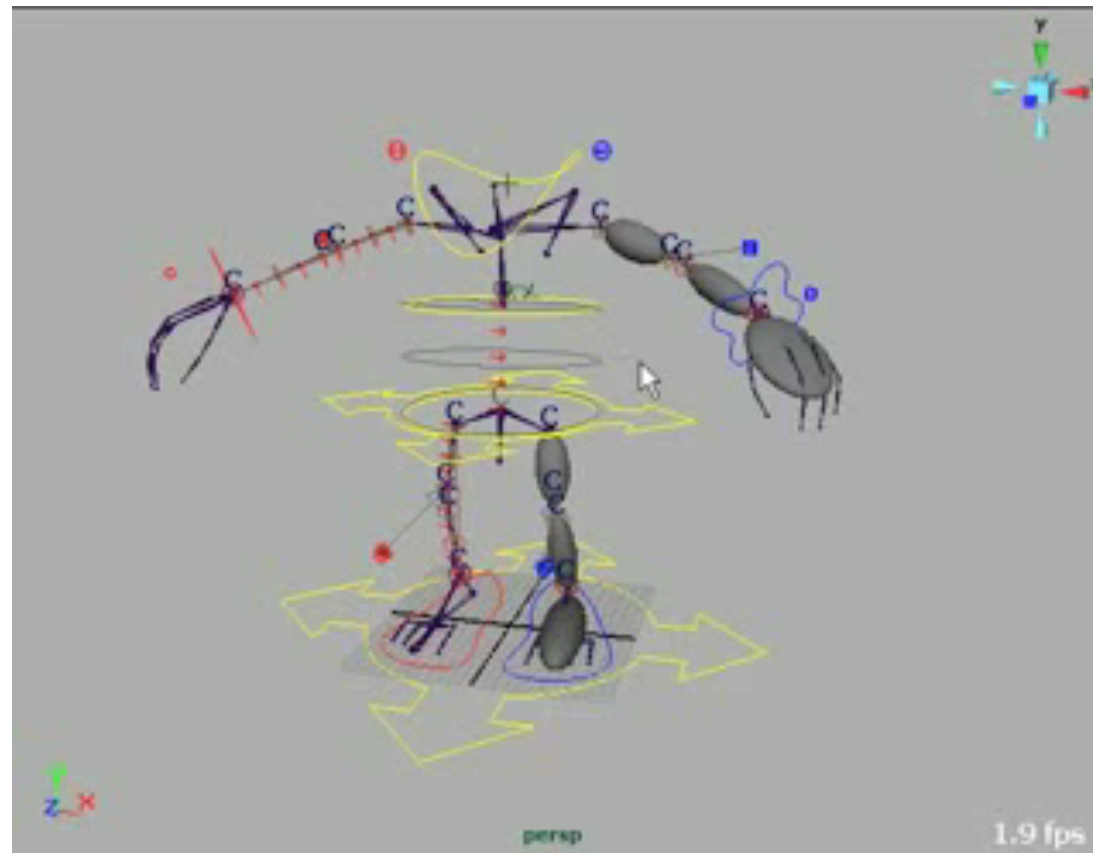
**kinematics** = motion of bodies **without** simulation of forces

**dynamics** = simulation/computation of forces and the motions of the objects resulting from them

- In computer graphics we always move within a continuum:



- Example of pure kinematics: *forward & inverse kinematics*



- Definition: a **particle system** is comprised of
  1. A set of **particles**; each particle  $i$  has, at least, the following attributes:
    - Mass, position, velocity ( $m_i, \mathbf{x}_i, \mathbf{v}_i$ )
    - Age  $a_i$
    - Force accumulator  $\mathbf{F}_i$
    - Optional: color, transparency, optical size, lifespan, type, ...
  2. A set of **particle sources**; each one is described by
    - Shape of the particle source
    - Stochastic processes that determine the initial attributes of the particles, e.g., velocity, direction, etc.
    - Stochastic processes that determine the number of particles created per frame
  3. Other (global) **parameters**, e.g.
    - TTL (time to live) = max. lifespan of particles
    - Global forces, e.g. gravitation, wind, ...
    - **The Algorithms**, that move and render of particles

- **Stochastic process =**
  - Simplest case: average + variance; process outputs random value according to uniform distribution
  - A bit more complicated: average and variance functions over time
  
- Remark on the geometric representation of particle sources:
  - Just an intuitive way to describe the stochastic process for the initial position of particles
  - Frequent forms: disk, cube, cone, etc.



- The "main loop" of a particle system:

```
loop forever:  
  render all particles  
   $\Delta t$  := rendering time  
  kill all particles with age > TTL (max. life-span)  
  create new particles at particle source  
  reset all force accumulators  
  compute all forces on each particle (accumulate them)  
  compute new velocities (one Euler step with  $\Delta t$ )  
  optionally modify velocities (*)  
  compute new positions (another Euler step)  
  optionally modify positions (e.g. b/c of constraints)  
  sort all particles by depth (for alpha blending)
```

## Remarks

- There is lots of space for optimizations, e.g.
  - Initialize force accumulators with gravitational force
  - don't increment the age of each particle "by hand"; instead, save the time of their creation in  $t_{\text{gen}}$ , then just test  $t_{\text{current}} - t_{\text{gen}} > \text{TTL}$ 
    - Will be important for parallel implementation later
- Remark on (\*) in the algorithm:
  - This is "non-physical", but allows for better kinematic control by the programmer/ animator
  - This is also necessary in case of collisions
- Often, we store a small history of the positions of particles, in order to create simple "motion blur" effects
- Particles can be killed by other constraints, too, e.g. distance from the source, entrance into a specific region, etc.
- For an efficient implementation, a "struct-of-array" data structure can be better! (SoA instead of AoS)

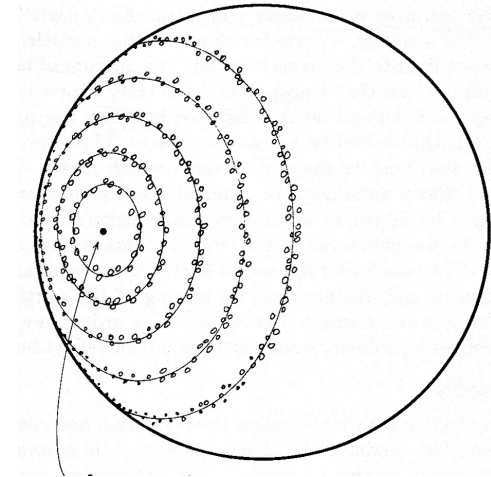
# Example of a Particle System

- Excerpt of "Wrath of Khan":

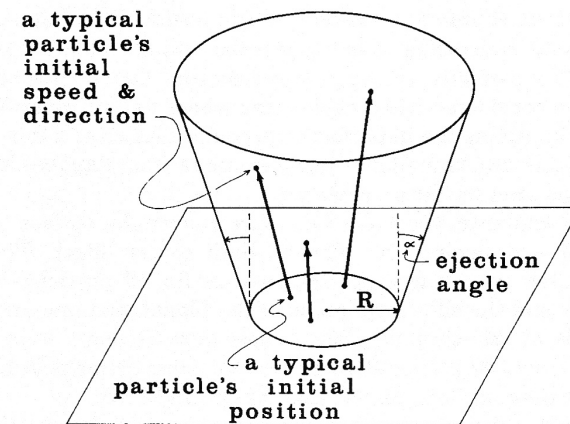


(Loren Carpenter, William Reeves, Alvy Ray Smith, et al., 1982)

- Particle source = circles on a sphere around the *point of impact*, which increase over time



- Stochastic processes for particle creation:
  - Capped cone normal to surface of sphere
  - Some variance of each particles lifespan



- Color =  $f(\text{age})$



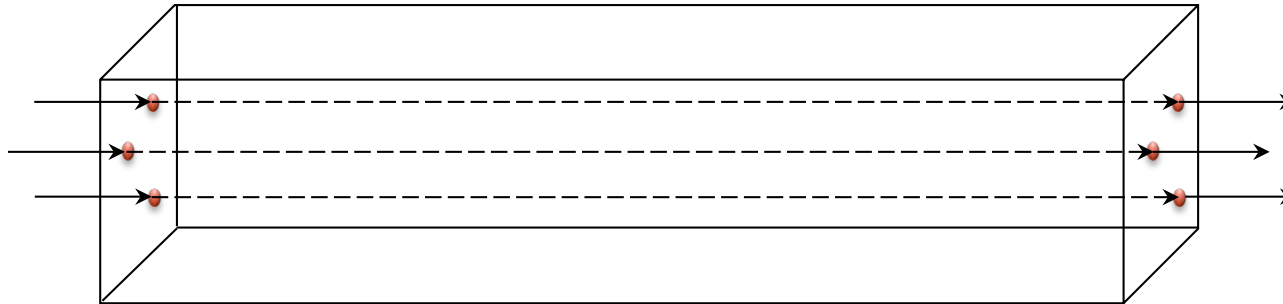
# Digression: the Panspermia Hypothesis



Karl Sims, 1990

# Operations on Particles

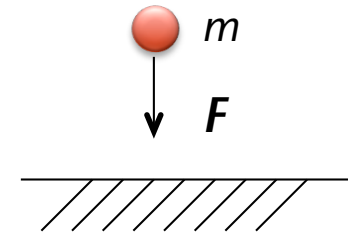
- Position operations:
  - Rather rare, e.g. “tunneling”



- Mostly done for collision handling

- Gravity:

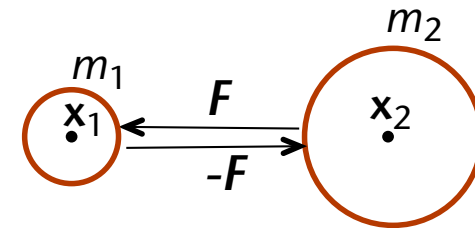
$$\mathbf{F} = m \cdot \mathbf{g} \quad , \quad g = 9.81 \frac{\text{m}}{\text{s}^2}$$



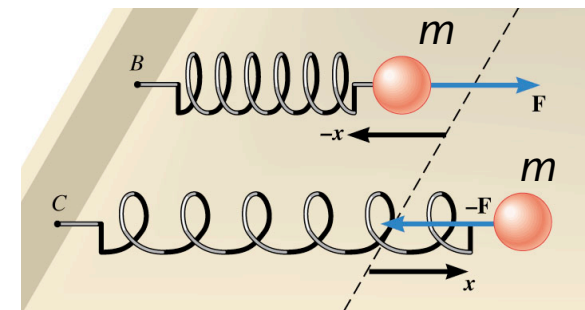
- Gravitation:

$$\mathbf{F} = G \cdot \frac{m_1 m_2}{r^2} \cdot \frac{\mathbf{x}_1 - \mathbf{x}_2}{|\mathbf{x}_1 - \mathbf{x}_2|}$$

$$G = 6,67 \cdot 10^{-11}$$



- Spring force: later



- **Viscous drag** (viskose Hemmung/Dämpfung):

$$\mathbf{F} = -b \mathbf{v}$$

in a stationary fluid/gas;

or, sometimes,

$$\mathbf{F} = 6\pi\eta r(\mathbf{v} - \mathbf{v}_{fl})$$

in fluids with velocity  $\mathbf{v}_{fl}$ , particles with radius  $r$ , viscosity  $\eta$ ;

or, sometimes, with high velocities

$$\mathbf{F} = -\frac{1}{2}c\rho A\mathbf{v}^2$$

$\rho$  = density,  $A$  = size of cross-sectional area,  $c$  = viscosity constant



- Electromagnetic force (Lorentz force):

$$\mathbf{F} = q \cdot \mathbf{v} \times \mathbf{B}$$

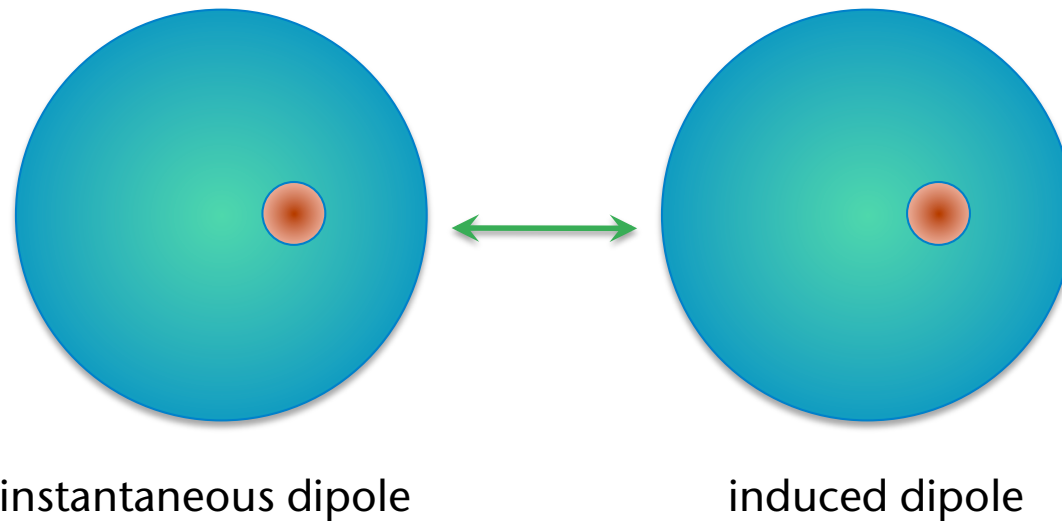
where  $q$  = charge of particle ,  $\mathbf{v}$  = velocity of particle,  
 $\mathbf{B}$  = magnetic field



[https://elearning.mat.univie.ac.at/physikwiki/index.php/LV002:LV-Uebersicht/Videos/Lorentzkraft\\_1](https://elearning.mat.univie.ac.at/physikwiki/index.php/LV002:LV-Uebersicht/Videos/Lorentzkraft_1)

# The Lennard-Jones Force

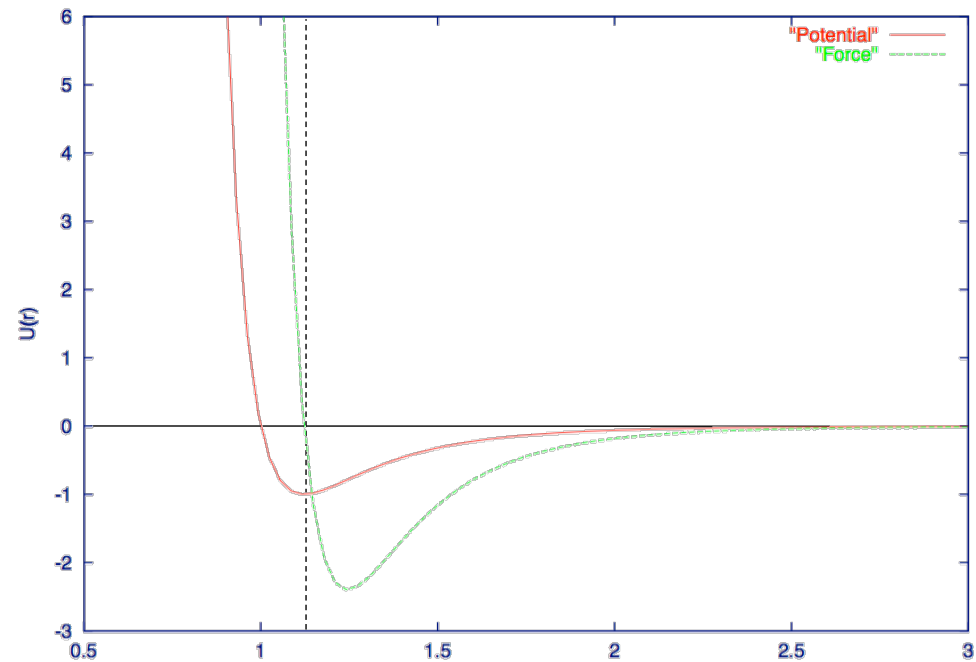
- There are two kinds of forces between atoms:
  - A repelling force (abstoßend) on short distances
  - An attracting force on mid-range distances (called *van-der-Waals* force or dispersion force)



- One (arbitrary) approximation of the Lennard-Jones force:

$$\mathbf{F} = \varepsilon \cdot \left( c \left( \frac{\sigma}{d} \right)^m - \left( \frac{\sigma}{d} \right)^n \right) \cdot \frac{\mathbf{x}_1 - \mathbf{x}_2}{\|\mathbf{x}_1 - \mathbf{x}_2\|}$$

where  $d = \|\mathbf{x}_1 - \mathbf{x}_2\|$   
 and  $\varepsilon, c, m, n$  are arbitrary constants (for our purposes)



# Non-Physical Effects

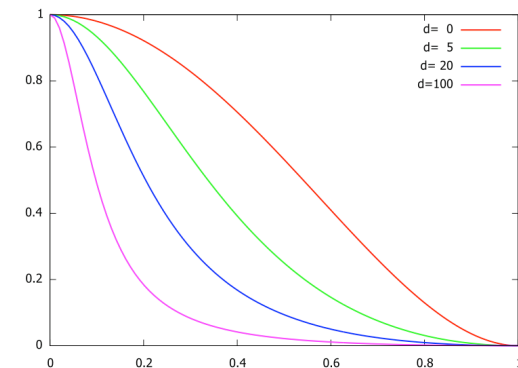
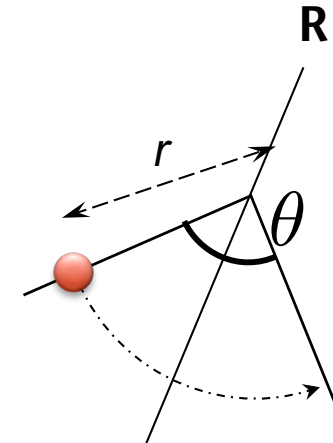
- **Vortex:** rotate particle about axis **R** with angle  $\theta = a \cdot f(r)$  where  $a =$  "force" of the vortex,  $r =$  distance particle  $\leftrightarrow$  axis, and

$$f(r) = \frac{1}{r^\alpha}$$

or

$$f(r) = \begin{cases} \frac{r^4 - 2r^2 + 1}{1 + dr^2} & , r \leq 1 \\ 0 & , r > 1 \end{cases}$$

- **Extensions:**
  - Take mass of particle into account
  - Use B-spline as axis of the vortex (e.g., for tornado)
  - Animate the axis of the vortex



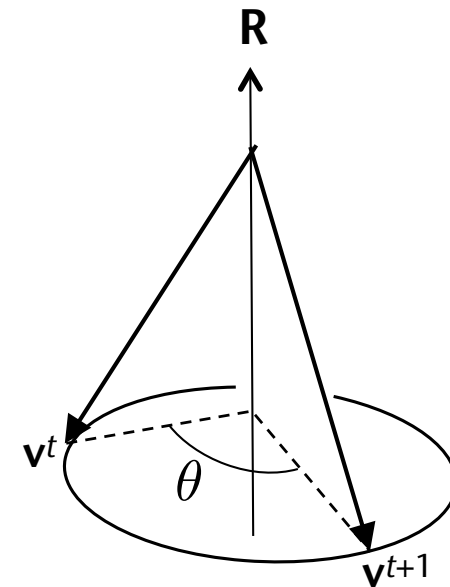
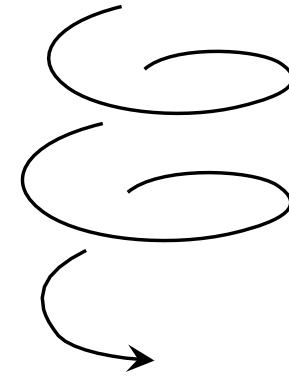


- Simple idea to make each particle move on its own individual spiraling path (e.g., for snow or smoke):

- Rotate  $\mathbf{v}$  (instead of the position) about an angle

$$\theta = \sigma \cdot \Delta t$$

- Again,  $\sigma$  can/should vary slightly and randomly, ditto the rotation axis
  - Also,  $\sigma$  and  $\mathbf{R}$  can be animated over time



- Most important kind of **geometric constraint**
- First, consider a collision with a plane

## 1. Collision check:

$$(\mathbf{x}^t - \mathbf{a}) \cdot \mathbf{n} > 0 \quad \wedge \quad (\mathbf{x}^{t+1} - \mathbf{a}) \cdot \mathbf{n} < 0$$

## 2. Collision handling: reflect $\mathbf{v}$

$$\mathbf{v}_N = (\mathbf{v} \cdot \mathbf{n}) \mathbf{n}$$

$$\mathbf{v}_T = \mathbf{v} - \mathbf{v}_N$$

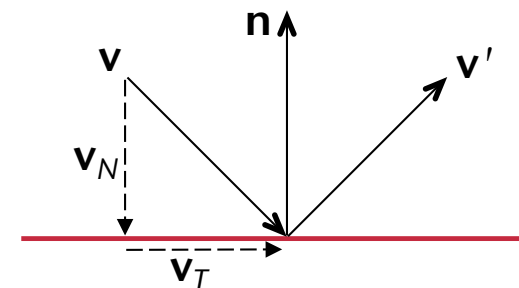
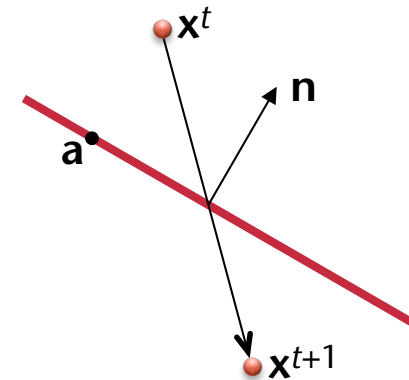
$$\mathbf{v}' = \mathbf{v}_T - \mathbf{v}_N = \mathbf{v} - 2(\mathbf{v} \cdot \mathbf{n}) \mathbf{n}$$

- Extension to friction and elastic/inelastic impact:

$$\mathbf{v}' = (1 - \mu) \mathbf{v}_T - \varepsilon \mathbf{v}_N$$

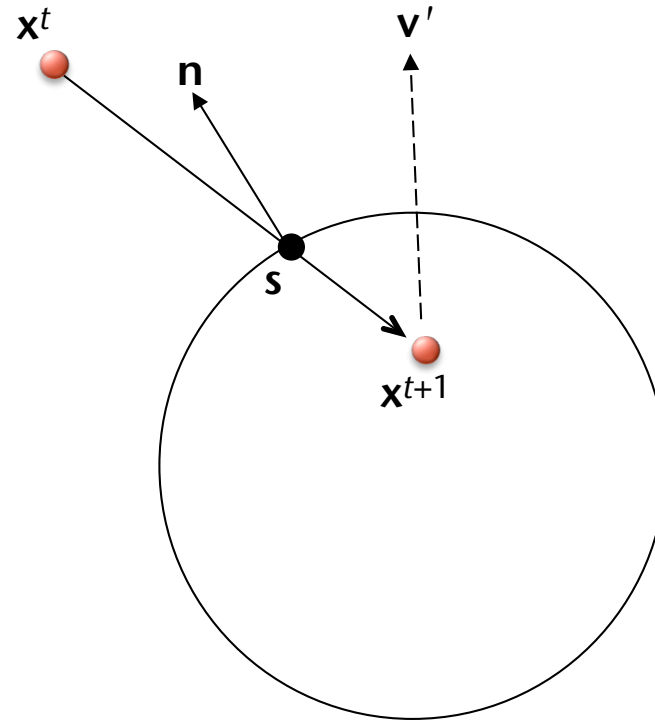
with  $\mu =$  **friction** parameter and

$\varepsilon =$  **resilience** (Federung / Elastizität)



- Collision with a sphere:
  - Compute exact intersection of  $x^t x^{t+1}$  with sphere  $\rightarrow s$
  - Determine normal  $n$  at point  $s$
  - Then continue as before
- Conclusion:
 

collision detection for particles =  
 "point inside geometry test", or,  
 more precisely: intersection test  
 between line segment and geometry

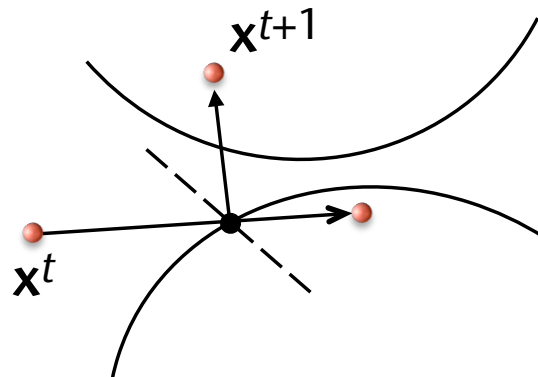


- For polyhedra and terrain: see "Computer Graphics 1"
- For implicit surfaces: see "Advanced Computer Graphics"

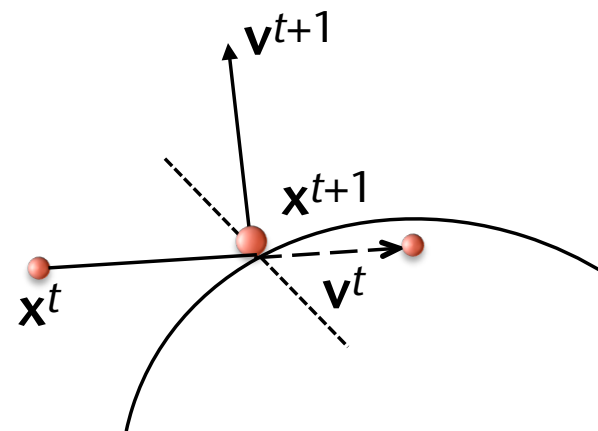
- Challenge: always create a *consistent* system after the collision handling!

- Problem: "double collisions" at narrow places

- Example:



- Better handling:

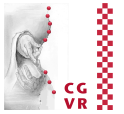


- There are more ways to handle these kinds of situations ...



Karl Sims: *Particle Dreams*

# Hierarchical Particle Systems

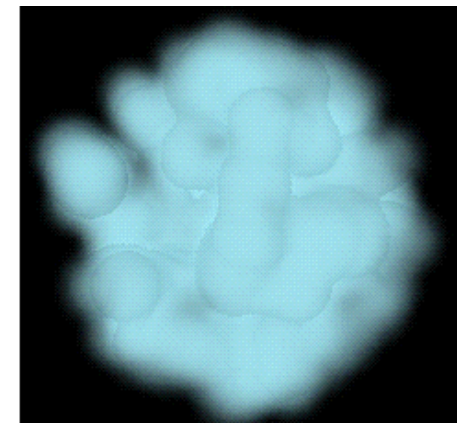
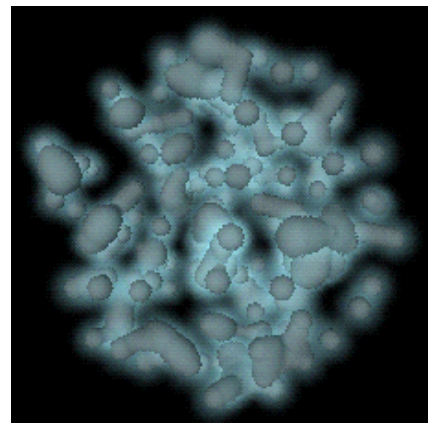
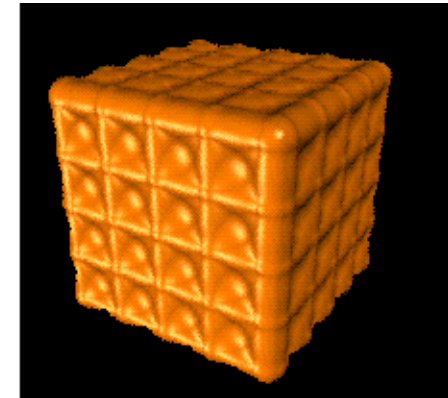
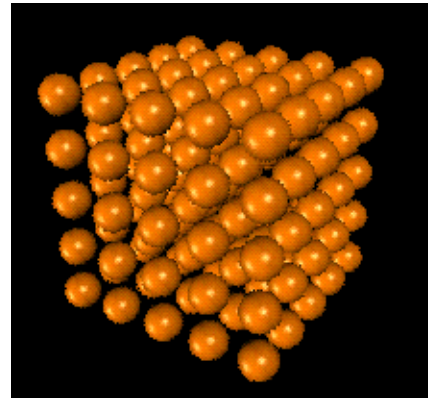


- Idea:
  - A particle represents a complete (lower level) particle system
  - Transformation of the parent particle moves the local coord frame of its ancillary particle system (just like with scenegraphs)
- Second-order particle systems:
  - All forces are being represented by particles
  - Forces can, thus, interact with each other, they can die, get born, etc.

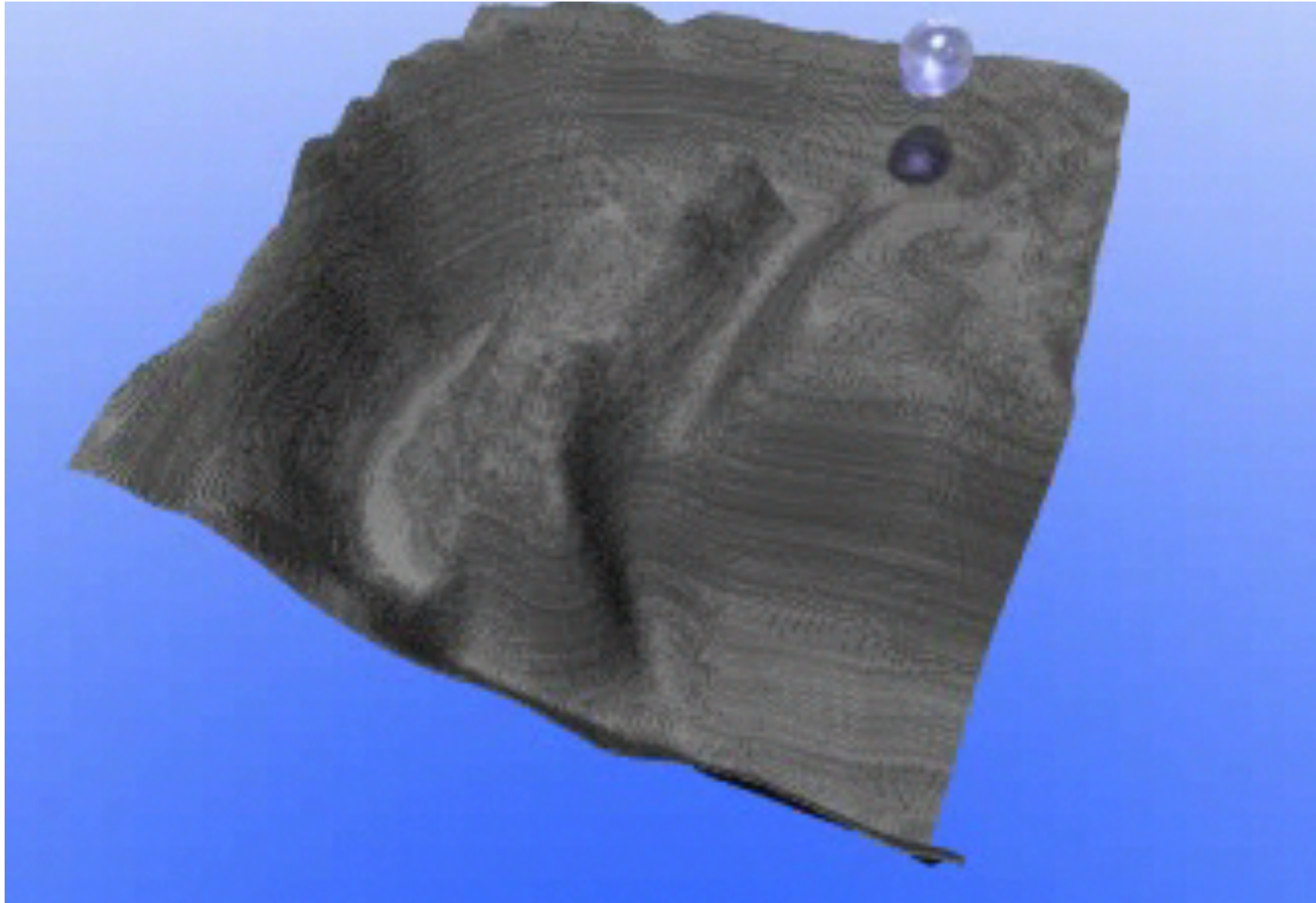


# Rendering Particles as "Blobby Objects"

- Regard particle as metaballs
  - In Advanced CG: *metaballs* = spheres that blend together to form (implicit) surfaces
  - Render using ray-casting
  - Either: find root of implicit surface
  - Or: accumulate the "density" along ray and interpret this as opacity or as luminance

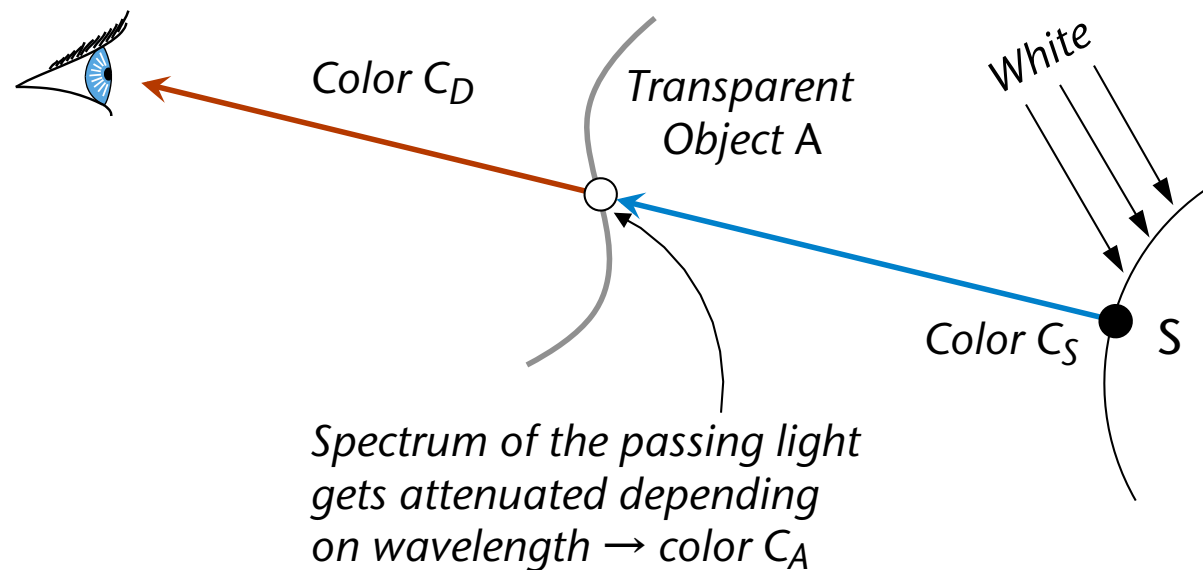


# Example



# Rendering Particles as Transparent Objects

- Render a small disc for each particle (*splat*, *sprite*, *billboard*)
- Often with transparency that decreases toward the rim
- Transparency  $\approx$  material that lets light pass partially
- Often, some wavelengths are attenuated more than others  $\rightarrow$  colored transparency
  - Extreme case: color filter in photography



- Approximation: **alpha blending**

- $\alpha \in [0, 1]$  = **opacity** (= opposite of transparency)

- $\alpha = 0 \rightarrow$  completely transparent,
    - $\alpha = 1 \rightarrow$  completely opaque

- "Color"  $C_A$  of object A = transmission spectrum  
(similar to reflectance spectrum of opaque objects, see CG1)

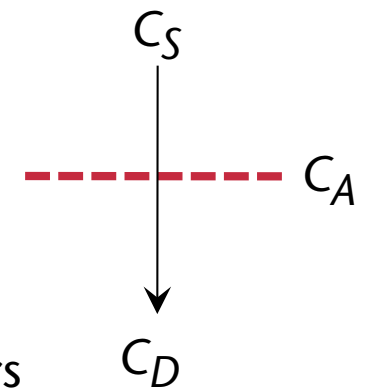
- Outgoing color:

$$C_D = \alpha C_A + (1 - \alpha) C_S$$

- Practical implementation:  $\alpha = 4^{\text{th}}$  component in color vectors

- During rendering, the graphics card performs these operations:

1. Read color from frame buffer  $\rightarrow C_S$
2. Compute  $C_D$  by above equation
3. Write  $C_D$  into framebuffer



- Problem: several transparent objects behind each other!

- Solution: first  $A$ , then  $B \rightarrow B$  gets killed by Z-test

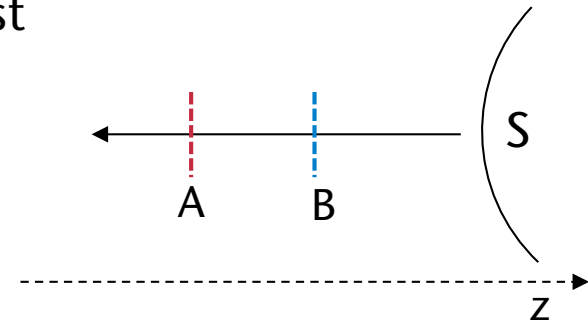
- Naïve idea: just switch Z-buffer off

1. First  $A$  then  $B$  (w/o Z-test) results in:

$$C'_D = \alpha_A C_A + (1 - \alpha_A) C_S$$

$$C_D = \alpha_B C_B + (1 - \alpha_B) C'_D$$

$$= \alpha_B C_B + (1 - \alpha_B) \alpha_A C_A + (1 - \alpha_B) (1 - \alpha_A) C_S$$



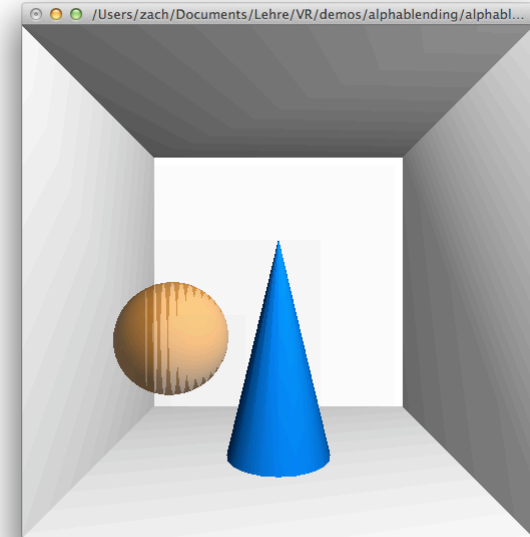
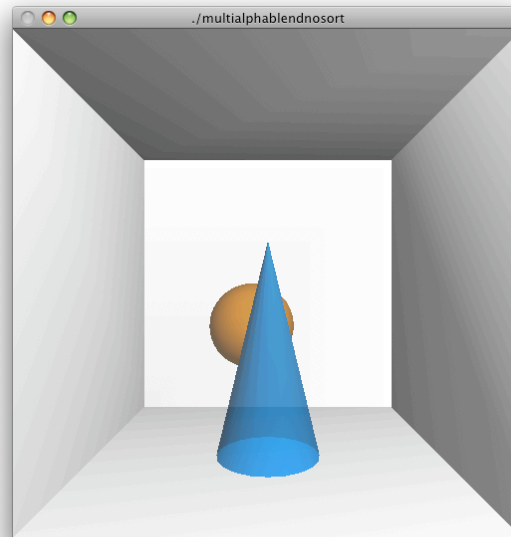
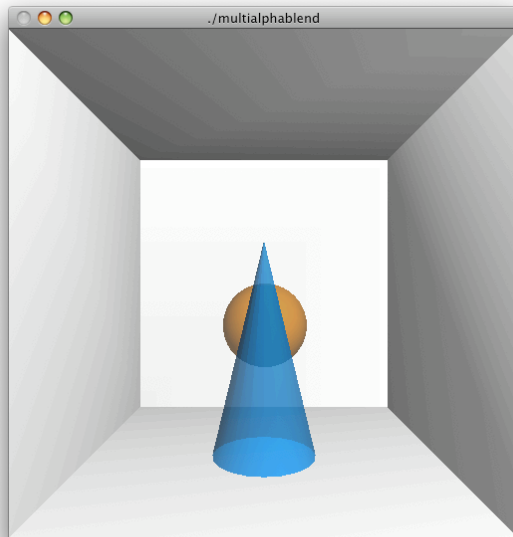
2. First  $B$  then  $A$  (w/o Z-test) results in:

$$C'_D = \alpha_B C_B + (1 - \alpha_B) C_S$$

$$C_D = (1 - \alpha_A) \alpha_B C_B + \alpha_A C_A + (1 - \alpha_B) (1 - \alpha_A) C_S$$

- Conclusion: **you must render transparent polygons/particles from back to front**, even if the Z-buffer is switched off!

- Examples (1 is correct, 2 with artifacts):





- In Open GL:
  - Switch blending on:

```
glEnable( GL_BLEND );
```

- Determine blending function:

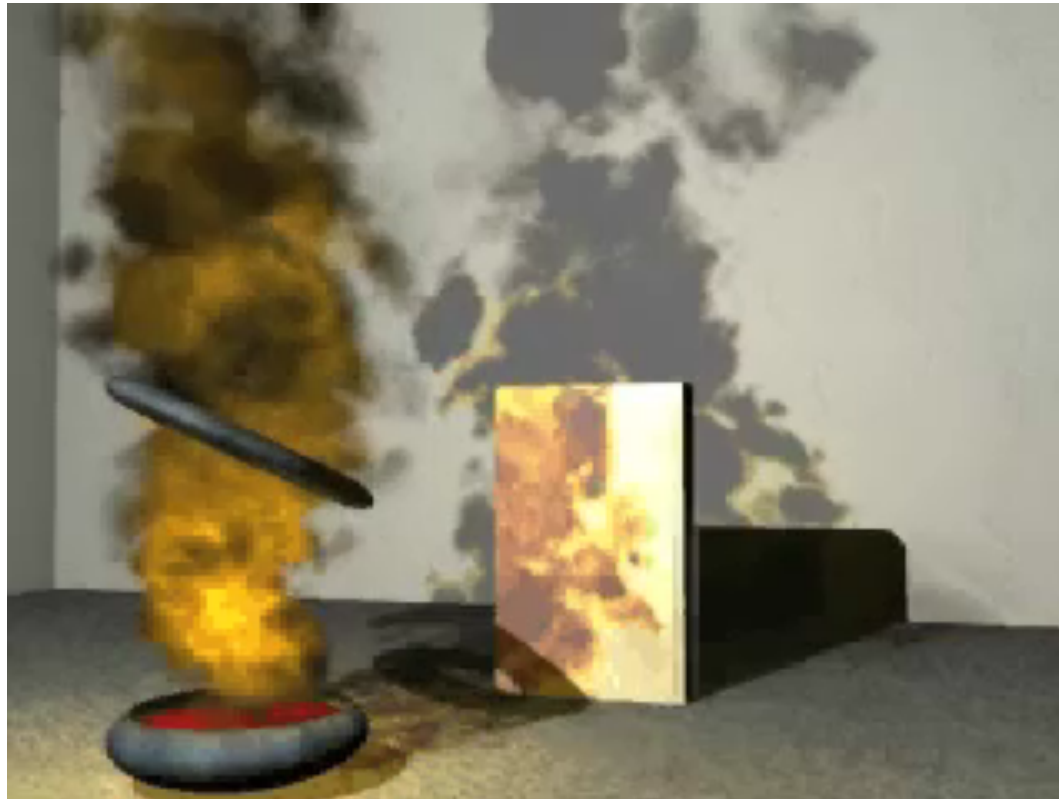
```
glBlendFunc( GLenum s, GLenum d );
```

`GL_SRC_ALPHA, GL_ONE_MINUS_SRC_ALPHA` →

$$C'_D = \alpha C_A + (1 - \alpha) C_D$$

where  $C_D$  = color from frame buffer;

- There are many more variants, e.g., you can just accumulate colors (`GL_ONE, GL_ONE`)



- Goals:
  1. Flames that look convincing
  2. Complete **control** over the flames
- The model:
  1. Represent individual flame (elements) by parametric curves → “spine” of a flame
  2. Regard the control points of the spine as particles
  3. Create surface around the spine where the burning happens
  4. Sample space in the proximity of the surface by "five" particles
  5. Render these particles (either volumetrically, or with alpha-blending)
- Controls for animators:
  - Length of spines (average & variance)
  - Lifespan of spine particles
  - Intensity of fire (=number of fire particles; particle sources, wind, etc)
  - Color and size of fire particles

- Creation of the *spines*:
  - Create a spine particle  $P$  in first frame
  - Simulate  $P$ : let it move upwards (buoyancy) and sideways (wind)

$$\mathbf{x}_P^{t+1} = \mathbf{x}_P^t + w(\mathbf{x}_P, t) + b(T_P) + d(T_P)$$

where

$w$  = wind field

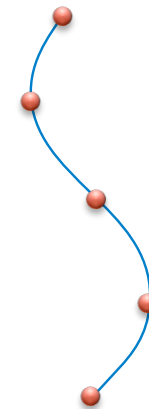
$b$  = buoyancy

$d$  = diffusion = noise;

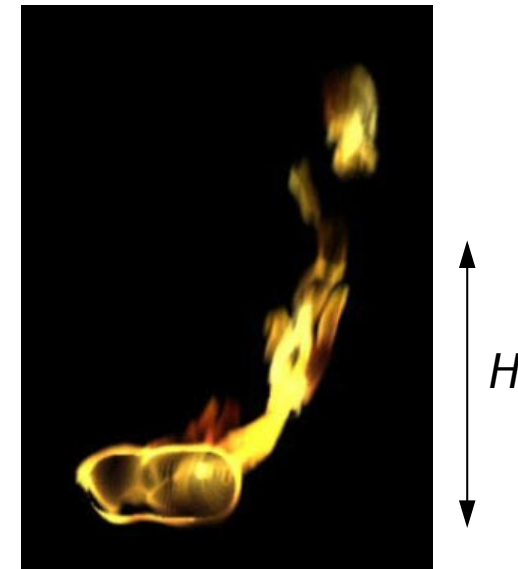
$T_P$  = temperature of particle = age

(Simplification here: particles don't have a mass)

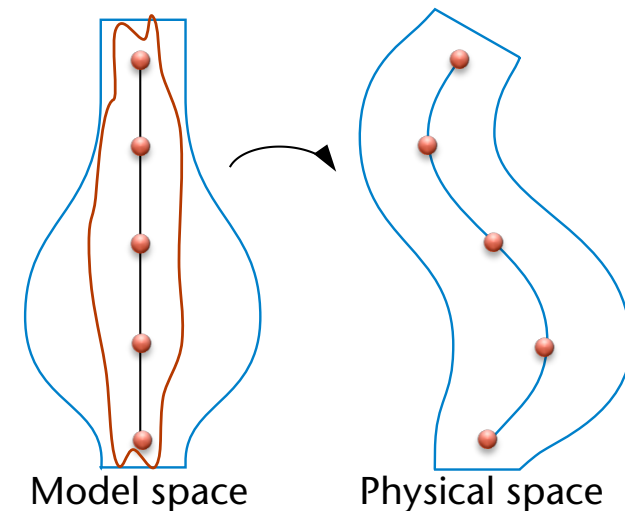
- In subsequent frames: create more particles; until max. number per flame is reached
- Connect all spine particles by B-spline



- At top of flames: break flame apart
  - Top part of spine is separated from rest at a random point in time, if height  $> H$
  - Lifespan after the split:
    - $\alpha^3$  sec , with random  $\alpha \in [0, 1]$



- The profile of a flame:
  - Rotationally symmetric surface around spine (= generalized cylinder)



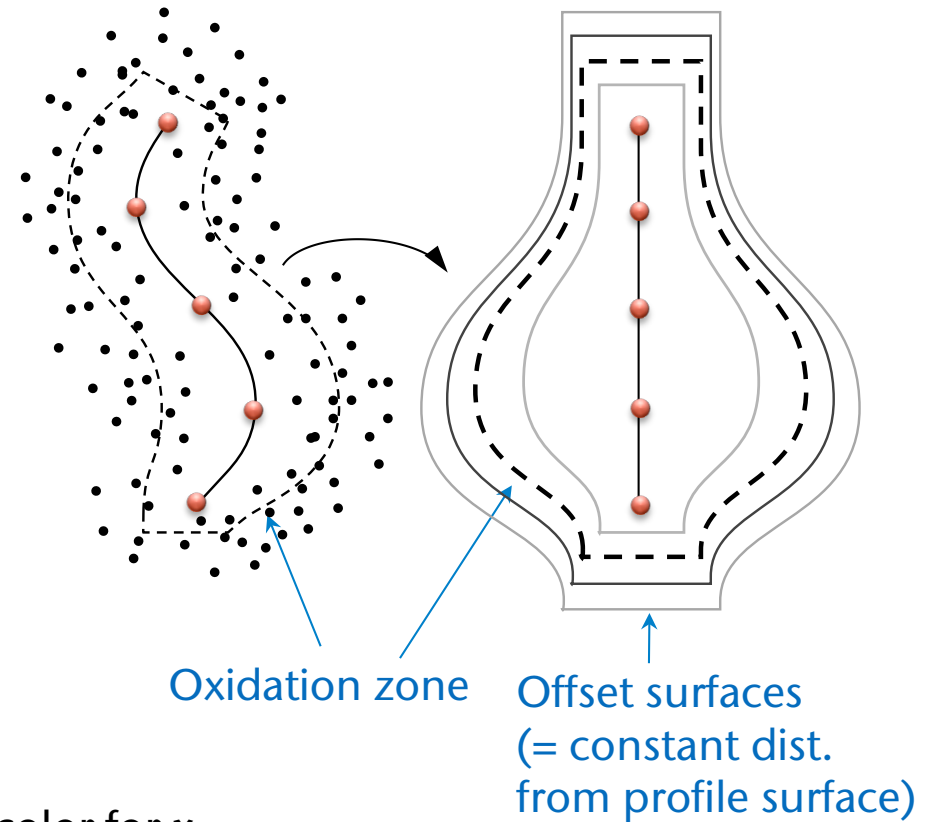
■ Rendering:

- Sample space around flame by a large number of "fire" particles according to this density function

$$D(\mathbf{x}) = \frac{1}{1 + \|\mathbf{x} - \mathbf{x}'\|}$$

where  $\mathbf{x}'$  = point on the (deformed) profile surface that is closest to  $\mathbf{x}$  :

1. Create random  $\mathbf{x}$
2. Transform into model space
3. Compute  $\mathbf{x}'$
4. Evaluate  $D$
5. If  $D(\mathbf{x}) >$  random number  $\rightarrow$  keep fire particle  $\mathbf{x}$
6. Texture profile surface with reference photo of real flame  $\rightarrow$  basis color for  $\mathbf{x}$



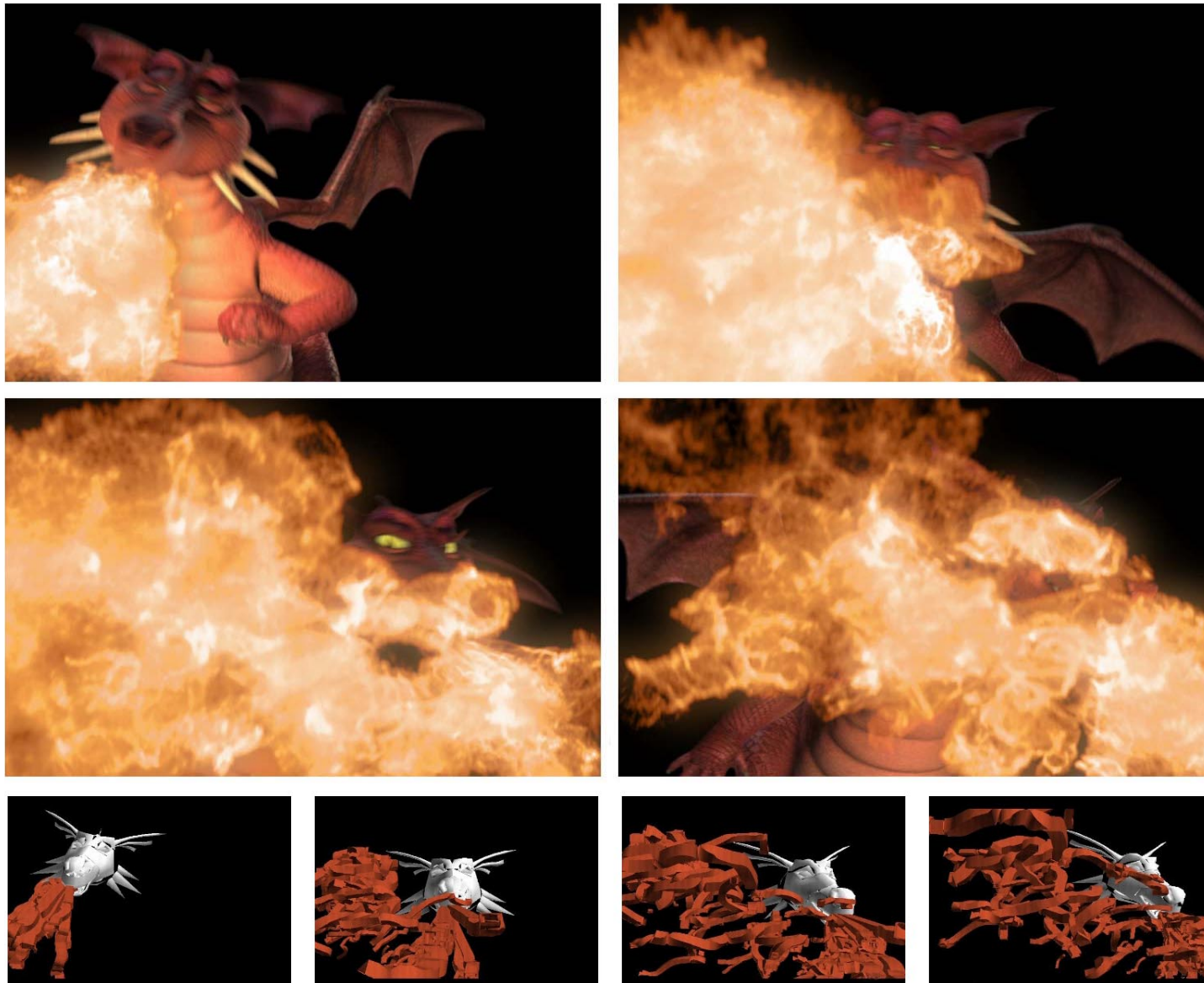


- Brightness of a fire particle at position  $\mathbf{x}$ :

$$E(\mathbf{x}) = k \frac{D(\mathbf{x})}{n}$$

where  $k$  = faktor for animator's control,  $n$  = number of samples

- Rule of thumb: ca. 10 samples per pixel, ca 10,000 samples per flame
- Discard samples on the inside of obstacles
- Smoke: render fire particles with height > "smoke height" in grey/black

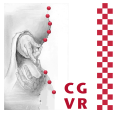


Arnauld Lamorlette and Nick Foster, PDI/DreamWorks





# Procedural Modeling of Plants with Particles



- Idea: use particles to simulate the transportation of water inside a leaf
  - Paths of particles constitute the vessels/"arteries" in the leaf
- Axioms:
  1. Nature always tries to minimize the total length of all arteries → particles will try to merge
  2. No water gets lost or gets added within the arteries →  
if 2 particles merge their paths, the resulting artery must have twice the cross-sectional area
  3. All arteries/paths emanate from the stem of the leaf

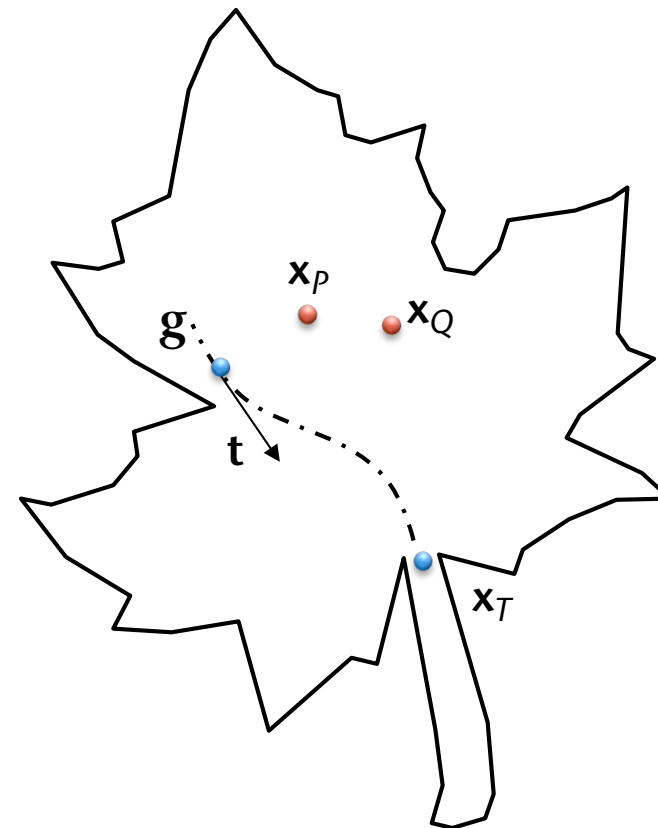


- Overview of the algorithm:

```
Initialize particles randomly on surface/rim of the leaf
loop until no particle is left:
    move each particle closer towards its nearest neighbor
      or towards an existig path,
      and in the direction of the stem
    if particle has reached the stem:
        kill it
    if two particles are "close enogh" to each other:
        merge both particles
```

# On the Motion/Simulation of Particles

- Let  $\mathbf{x}_P$  = current position of particle  $P$
- $\mathbf{x}_T$  = target position (stem of leaf)
- $\mathbf{g}$  = point on an existing path closest to  $\mathbf{x}_P$
- $\mathbf{t}$  = tangent in  $\mathbf{g}$  (normalized)
- $\mathbf{x}_Q$  = particle closest to  $P$



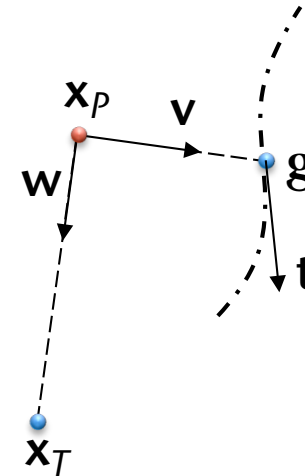


- If  $\|\mathbf{x}_P - \mathbf{g}\| < \|\mathbf{x}_P - \mathbf{x}_Q\|$  :

- Let:

$$\mathbf{v} = \frac{\mathbf{g} - \mathbf{x}_P}{\|\mathbf{g} - \mathbf{x}_P\|}$$

$$\mathbf{w} = \frac{\mathbf{x}_T - \mathbf{x}_P}{\|\mathbf{x}_T - \mathbf{x}_P\|}$$



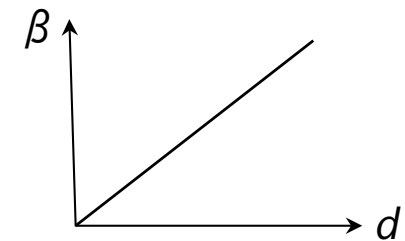
- New position:

$$\mathbf{x}'_P = \mathbf{x}_P + \alpha \mathbf{w} + (1 - \alpha) (\beta \mathbf{v} + (1 - \beta) \mathbf{t})$$

with

$$\beta = \beta (\|\mathbf{x}_P - \mathbf{g}\|)$$

- If  $\beta$  is (approximately) linear, this will yield particle paths, that are tangential to existing paths, and perpendicular to them when further away



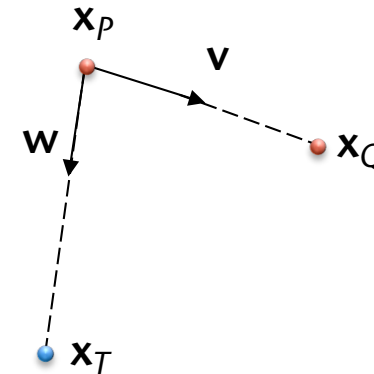
- Else (i.e.,  $\|\mathbf{x}_P - \mathbf{x}_Q\| < \|\mathbf{x}_P - \mathbf{g}\|$ ) :

- Let

$$\mathbf{v} = \frac{\mathbf{x}_Q - \mathbf{x}_P}{\|\mathbf{x}_Q - \mathbf{x}_P\|}$$

- New position:

$$\mathbf{x}'_P = \mathbf{x}_P + \gamma \mathbf{v} + (1 - \gamma) \mathbf{w}$$

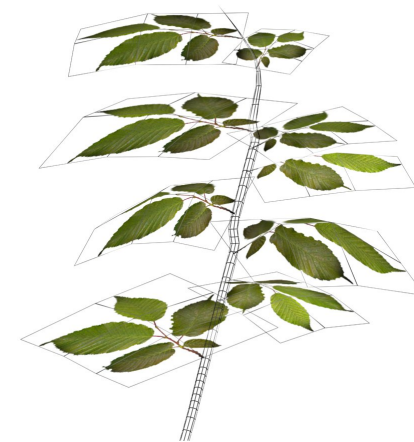
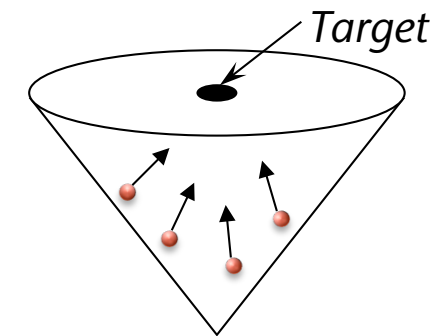
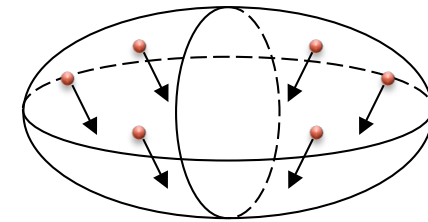


- About the thickness of the arteries:

- Each particle has size = size of cross-sectional area of artery
  - At beginning: each particle has unit size
  - In case of merging: add sizes
  - In case of particle hitting existing path: add size of particle from there on until the stem (target position)

# Modeling of Trees

- Works exactly the same
- Input from the animator: geometry of tree crown (= particle source)
  - Create particles within the volume by stochastic process
- Create geometry of branches & twigs by sweeping a disk along the path
- Place leaf primitives at end of twigs



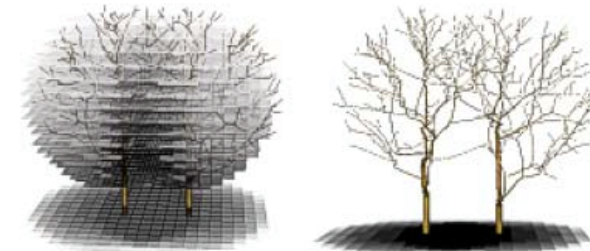
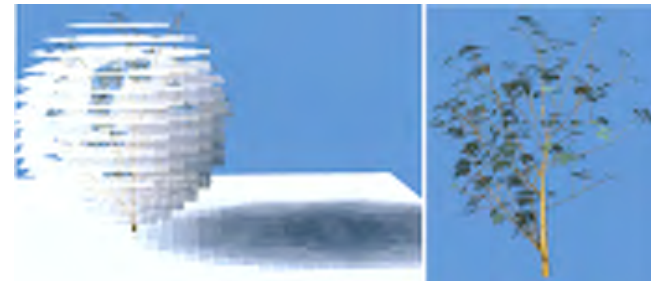
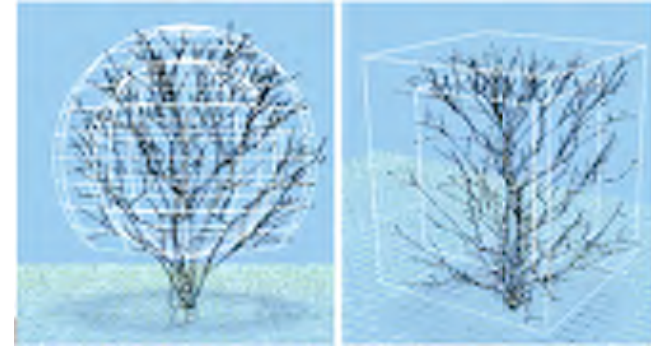
- Example of the procedural modeling process:





## Incorporation of Lighting Conditions

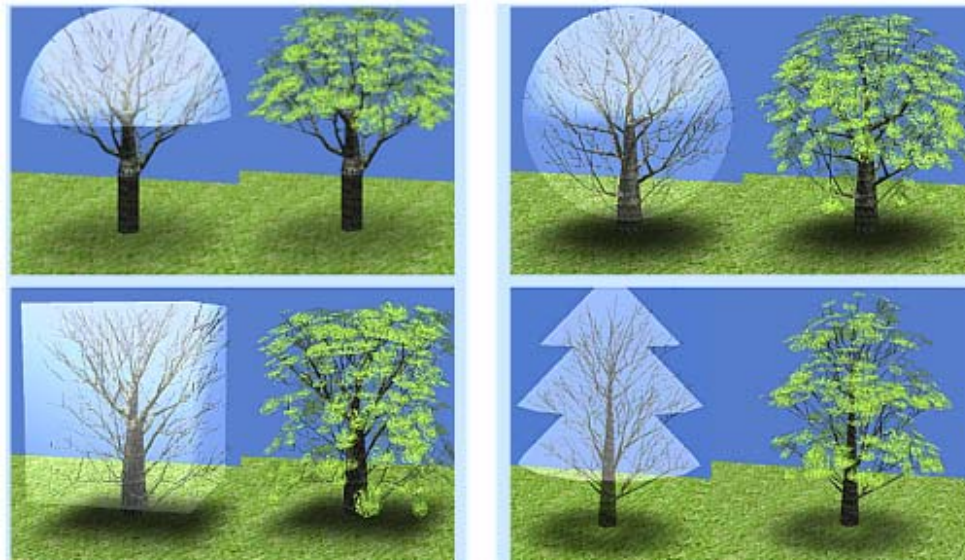
- Observation: regions with less light irradiation have less branches/leaves
- Can be modeled relatively easy:
  - Put tree inside 3D grid
  - Approximate the (not yet existing) foliage by a spherical or cubical shell
  - Compute light irradiation for each grid node by casting a ray outward
  - During particle creation: modify probability of creation according to irradiation (obtained by trilinear interpolation of grid nodes)







# Examples





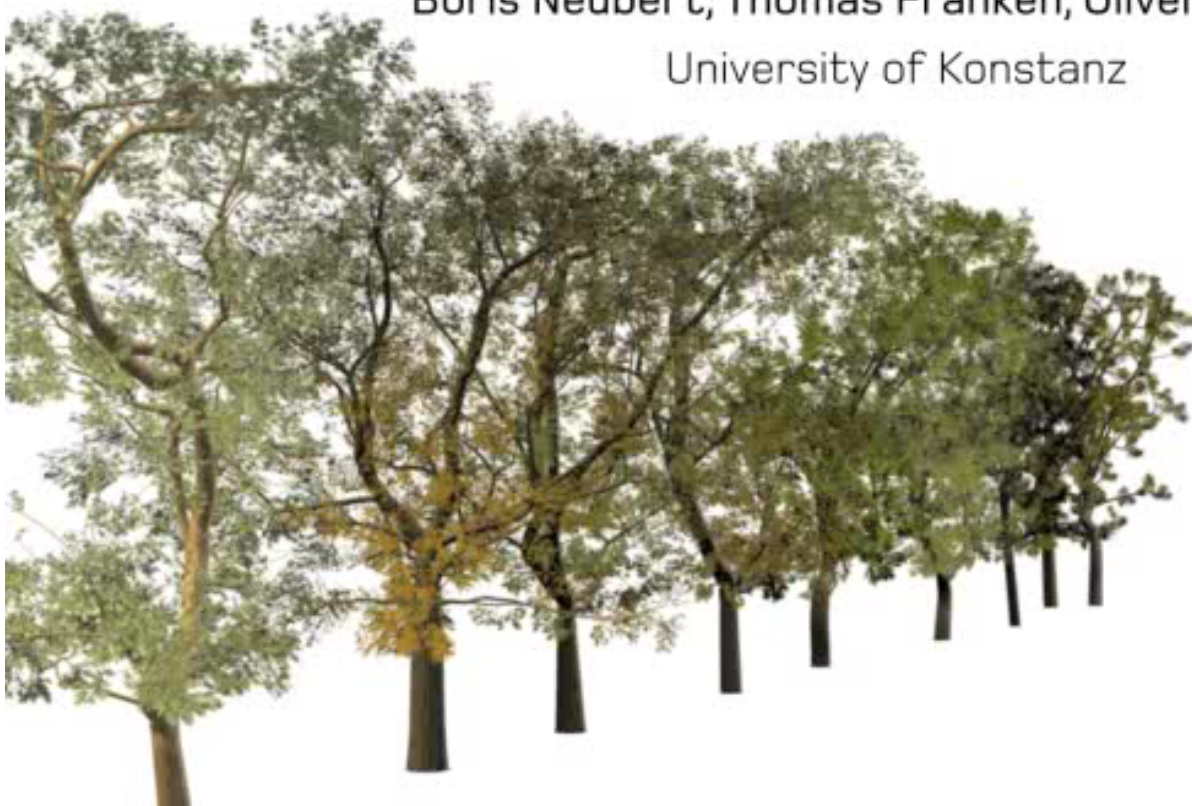






## Approximate Image-Based Tree Modeling using Particle Flows

Boris Neubert, Thomas Franken, Oliver Deussen  
University of Konstanz





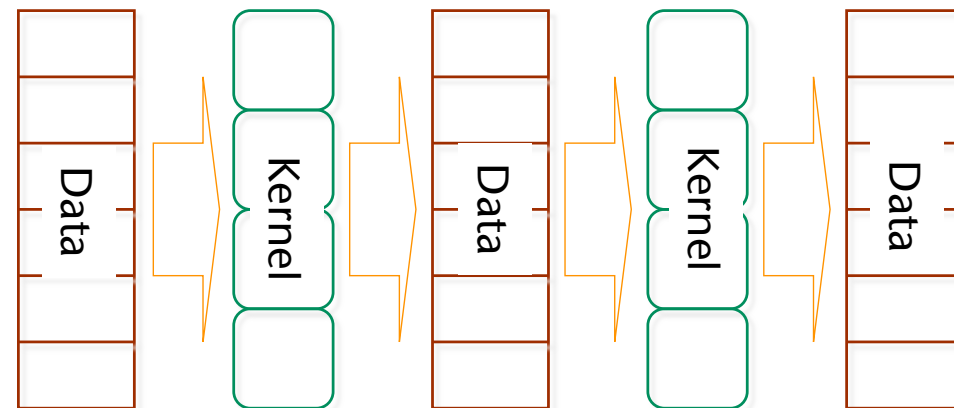
The Adventures of André and Wally B. (Pixar, 1984)

# Massively-Parallel Simulation on Stream Architectures

- Background on streaming architectures (and GPUs):
  - **Stream Programming Model** =  
 "Streams of data passing through computation kernels."
  - **Stream** = ordered, homogenous set of data of arbitrary type
  - **Kernel** = program to be performed on *each* element of the input stream
- Sample stream program:

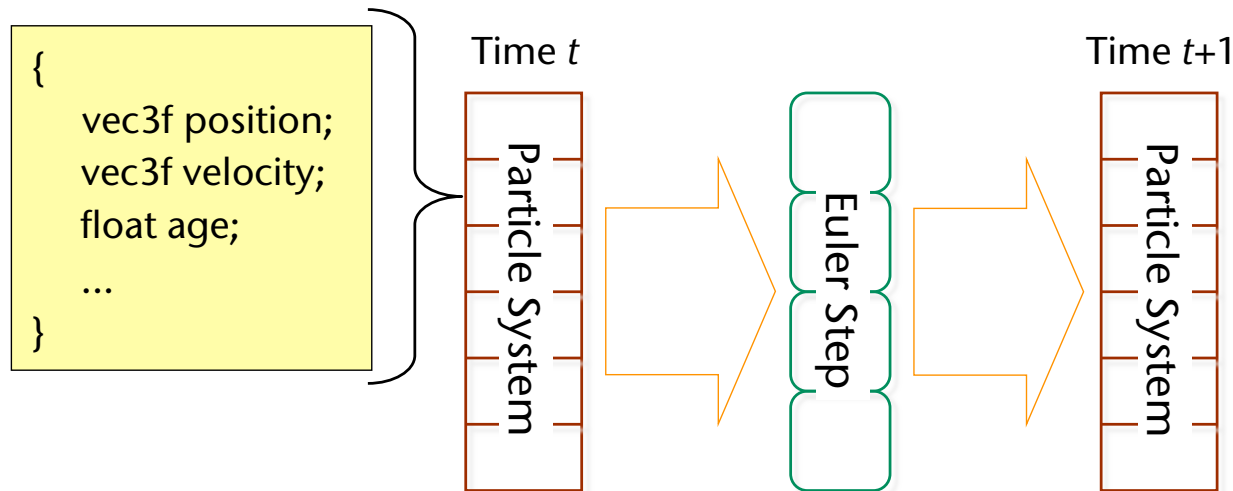
```

{
  stream A, B, C;
  ...
  kernelfunc1( input: A,
              output: B);
  kernelfunc2( input: B,
              output: C);
  ...
}
    
```



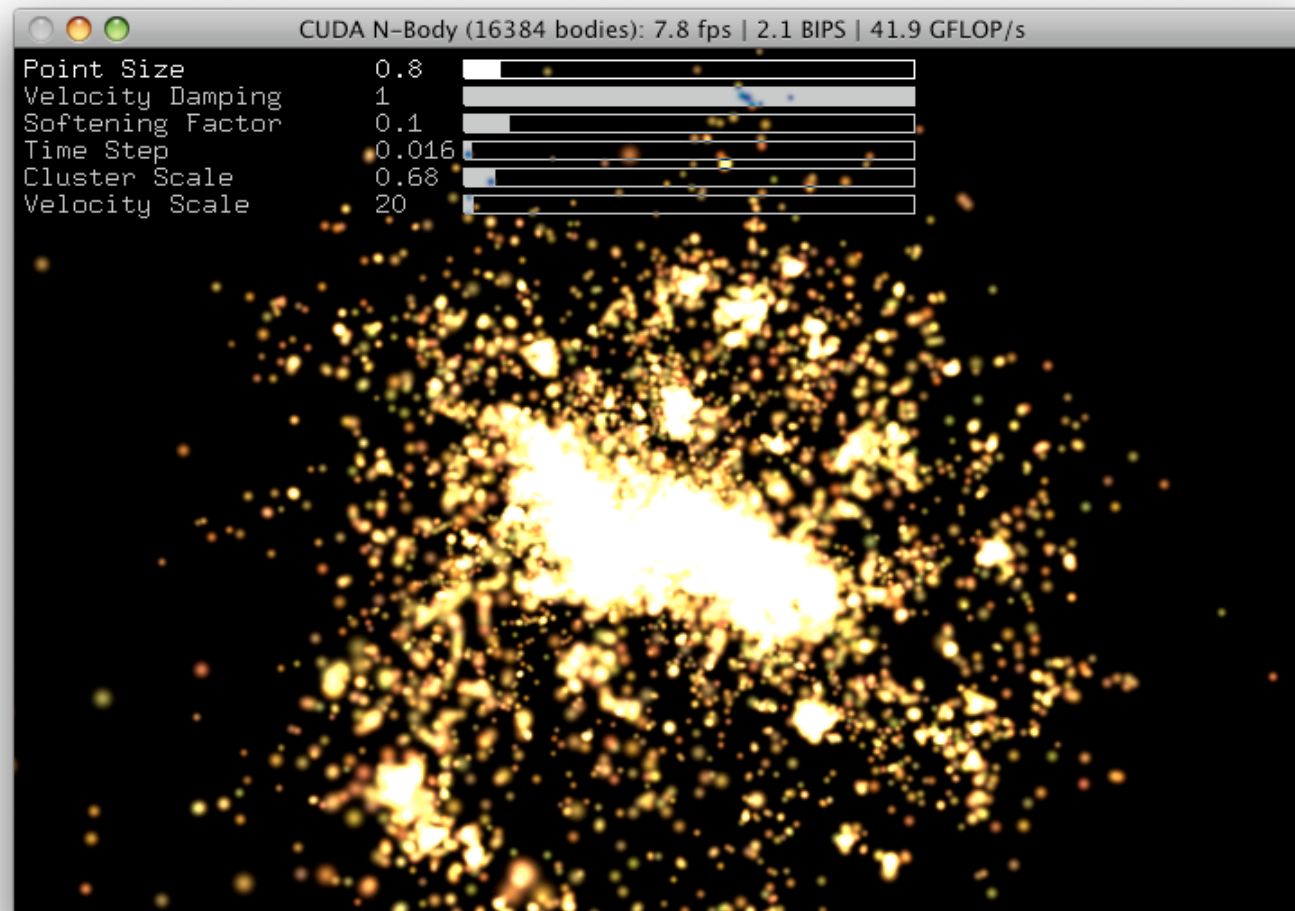
- Today's GPU's are streaming architectures, i.e., massively-parallel , general purpose computing architectures

- Today's GPU's have — at least conceptually — 1000's of processors
- Each processor (kernel) can read several (a few) elements from the input stream, but it can/should write **only one** output element!
- Particle Simulation on GPU's:



- For more teaching on massively parallel algorithms  
→ my Massively-Parallel Algorithms course 😊





N-body simulation

<http://www.nvidia.com/cuda>

